mination of the discontinuity parameters. It may be observed that the resonance curves pass close to, rather than through, the "fixed points" computed from calibration measurements. This may be attributed to the finite thickness of the discontinuity.

From each point $\tilde{D}$, $\tilde{S}$ on the resonance diagram, a point Σ, Δ on the tangent curve for the network in the discontinuity plane may be computed via (3) and (11). Fig. 11 shows this result for points chosen from all the curves of Fig. 9; only data from the immediate neighborhood of the "fixed points" have been excluded because of the thickness effect noted previously. The final test of the internal consistency of the two-mode dai was a precision analysis of the tangent curve,[7] whic will be described briefly. Values Δ' corresponding 1 measured values of Σ were computed from the tangei relation employing the parameters Σ_0, Δ_0, γ listed c Fig. 11. The differences $(\Delta'-\Delta)$ between the measure and computed values are a measure of the accurac with which the parameters Σ_0, Δ_0, γ represent the e: perimental data or the internal consistency. For tl data presented, the measure of accuracy, $|\Delta'-\Delta|/\lambda$ <0.005, is comparable to values attained in single moc precision measurements.

Mode Couplers and Multimode Measurement Techniques*

D. J. LEWIS†

Summary—**The measurement of harmonic and spurious signals in waveguide systems is complicated by the fact that one must usually deal with a multimodal measurement. Since the energy may propagate in any mode consistent with the frequency and waveguide geometry, the measurement system used must discriminate between these different modes.**

A simple and direct approach to this problem is through the use of "mode couplers" which couple selectively to any desired mode. Theoretical and practical details for mode couplers for the first five modes in rectangular waveguide are presented, as well as the application and limitations of this measurement technique.

Introduction

The measurement of spurious and harmonic signals in waveguide systems is usually complicated by the fact that one must work with a multimode system. The energy one wishes to measure can be propagated in every mode consistent with the frequency and waveguide geometry. For example, a 6000-mc signal in a standard S-band waveguide ($3\times 1\frac{1}{2}$ inches) can travel in the TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11} modes. The distribution of power among these modes will, of course, depend on how the guide is excited.

To specify more exactly the nature of the problem, assume that the total power carried in an n-mode system is to be measured. To solve this problem n couplers are required, the output of each coupler being a linear function of the amplitude of each mode. Thus:

$$E = k_1M_1 + k_2M_2 + \cdots k_nM_n. \qquad (1)$$

A series of n amplitude and phase measurements usir these couplers will therefore yield the following set (equations:

$$\begin{aligned} E_1 &= k_{11}M_1 + k_{12}M_2 + \cdots k_{1j}M_j + \cdots k_{1n}M_n \\ E_2 &= k_{21}M_1 + k_{22}M_2 + \cdots \\ &\vdots \\ E_r &= k_{r1}M_1 \quad \cdots \quad k_{rj}M_j \\ &\vdots \\ E_n &= k_{n1}M_1 \quad \cdots \quad k_{nn}M_n. \end{aligned} \qquad ($$

Theoretically it should be possible to solve for tl strength of the different modes from these equatioı but clearly, where more than two or three modes a involved, the solution of such a set of complex equatioı will be a tedious task. An alternative is to reduce tl value of the cross-coupling terms k_{rj} to a value so lo that the output of each coupler is essentially dependeı on only one mode. This approach also greatly simplifi the measurement technique since the need for pha: measurements is eliminated.

Mode Couplers

The basic mode coupler is shown in Fig. 1. It consis of two parallel waveguides mutually coupled throug two small apertures. The signal to be measured traveling from left to right in the lower or primaı waveguide. The upper or secondary waveguide coı

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tains a probe to extract the coupled power and a termination to eliminate reflections. The dimensions of the secondary waveguide are such that at the signal frequency only the TE_{10} mode can propagate.

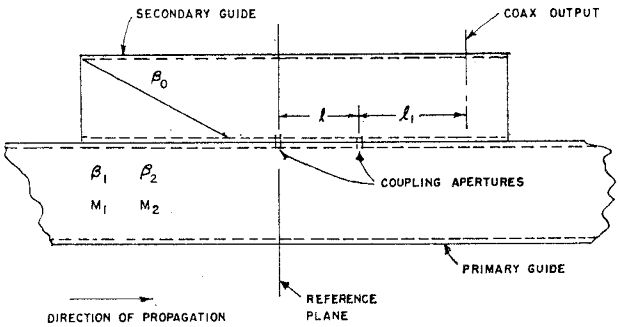

Fig. 1—Basic mode coupler.

For the sake of simplicity the discussion will be restricted to the case of two modes of propagation having field strengths M_1 and M_2 and phase constants β_1 and β_2, respectively. The phase constant of the TE_{10} mode in the secondary guide is β_0. The coupling apertures are identical and have field coupling coefficients k_1 for the M_1 and k_2 for the M_2 mode. The aperture spacing is l.

To develop the coupling equations, we assume a reference plane at the first aperture. Omitting the phase shift in the apertures, the forward coupled wave measured at the probe which results from M_1 is then given by:

$$E_{f1} = k_1 M_1 [e^{-j\beta_1 l} + e^{-j\beta_0 l}] e^{-j\beta_0 l_1} \quad (3)$$

or

$$|E_{f1}| = 2k_1 M_1 \cos \frac{(\beta_1 - \beta_0)l}{2}. \quad (4)$$

Similarly E_{f2} the forward wave coupled from M_2 is:

$$E_{f2} = k_2 M_2 [e^{-j\beta_2 l} + e^{-j\beta_0 l}] e^{-j\beta_0 l_1} \quad (5)$$

and

$$|E_{f2}| = 2k_2 M_2 \cos \frac{(\beta_2 - \beta_0)l}{2}. \quad (6)$$

Our objective is to design the system so that the probe output will be a measure of the strength of either M_1 or M_2 with a minimum of interference from the other. If we wish to sample M_1, the difference between the coupled waves is maximized by establishing the conditions:

$$E_{f1} = 2k_1 M_1 \quad (7a)$$

$$E_{f2} = 0. \quad (7b)$$

This requires that:

$$(\beta_1 - \beta_0)l = 2n\pi \quad (8a)$$

$$(\beta_2 - \beta_0)l = (2k + 1)\pi \quad (8b)$$

$$n = 0, 1, 2, 3 \cdots$$

$$k = 0, 1, 2, 3 \cdots.$$

Eq. (8a) and (8b) must be satisfied simultaneously to meet the imposed conditions. Taking the ratio of these two equations, one can develop the necessary relations between the phase constants of the two modes in question and the phase constant β_0 of the secondary waveguide as follows:

$$\frac{(\beta_1 - \beta_0)}{(\beta_2 - \beta_0)} = \frac{2n}{2k + 1} \quad (9)$$

$$n = 0, 1, 2, 3 \cdots$$

$$k = 0, 1, 2, 3 \cdots.$$

The behavior of this ratio can be readily visualized with the aid of the normalized phase constant plots given in Fig. 2. It can be seen that for a given f_0, to obtain the correct ratio for (9) one must pick the appropriate value of f_{c0} (the cutoff frequency of the TE_{10} mode in the secondary guide). If l is then selected to satisfy either (8a) or (8b) (the other of course is automatically satisfied) the coupler will have the desired characteristic of rejecting M_2 while giving maximum response to M_1.

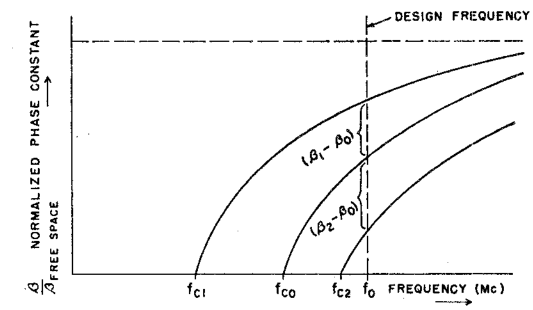

Fig. 2—Normalized plots of phase constant as a function of frequency for three modes having cutoff frequencies f_{c0}, f_{c1}, and f_{c2}.

The values selected for n and k in (9) are quite arbitrary. However, large values of n and k lead to greater aperture spacing. Since this results in smaller bandwidths as well as greater physical dimensions, it is advantageous to pick small numbers. A ratio of 2/3 in (9) was found satisfactory experimentally.

If β_0 is made equal to β_1, then from (4) the coupling to M_1 will depend only on k. In this case, l is simply dimensioned to satisfy (8b). This would not be a practical design for the TE_{10} mode, however, since the secondary guide would have the same dimensions as the primary guide. The secondary guide itself would then be capable of supporting each of the higher order modes associated with the primary guide.

It is evident from Fig. 2 that (8a) and (8b) can be satisfied only at the design frequency, f_0. The variation of coupling with frequency is easy to picture, however, since the arguments of the cosine functions in the coupling equations [(4) and (6)] are proportional to the differences between the curves in Fig. 2. These differences are plotted in Fig. 3 while the coupling corresponding to these curves is shown in Fig. 4.

The mode selectivity can be defined as the difference in coupling between the desired and undesired modes.

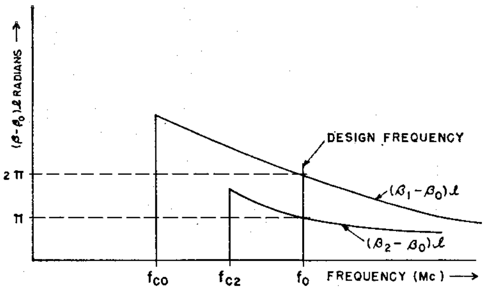

Fig. 3—Variation of $(\beta_1-\beta_0)l$ and $(\beta_2-\beta_0)l$ with frequency.

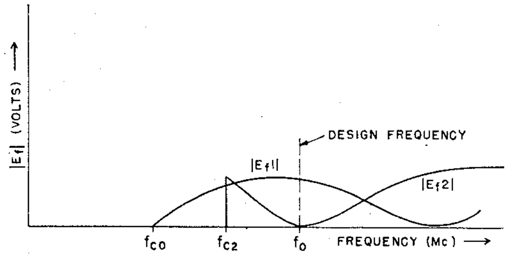

Fig. 4—Output as a function of frequency for the two modes, M_1 and M_2.

Therefore:

$$S = 20 \log \frac{\dfrac{|E_{f1}|}{M_1}}{\dfrac{|E_{f2}|}{M_2}}. \tag{10}$$

Substituting from (4) and (6):

$$S = 20 \log \left[\frac{\cos \dfrac{(\beta_1 - \beta_0)l}{2}}{\cos \dfrac{(\beta_2 - \beta_0)l}{2}} \right] + 20 \log \left(\frac{k_1}{k_2} \right). \tag{11}$$

Typical curves of S are shown in Fig. 5.

The second term in (11) is the contribution to the mode selectivity which arises from the difference in coupling of two different modes to a given aperture. For example, a narrow slot will couple strongly to any mode which has a magnetic field component parallel to the slot and weakly to modes whose H fields are perpendicular to the slot. This approach has been investigated by Judy and Angelakos [3] in connection with the development of mode selective directional couplers. Their results were used extensively in the design of the coupling aperture of the mode couplers.

The total mode selectivity, then, is a function of two different mechanisms, one depending on the field configuration of the mode, and the other on the phase constant or velocity with which the mode propagates. By choosing the correct combination of aperture geometry, aperture spacing, and secondary waveguide dimensions, one may achieve a considerable degree of mode selectivity.

For example, suppose a pair of longitudinal slots in

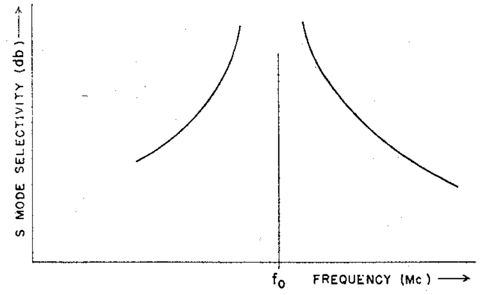

Fig. 5—Mode selectivity as a function of frequency.

the center of the broad face of the waveguide are used as the coupling apertures. Because of their location at a point of minimum longitudinal magnetic field, the slots will not couple to TE_{10}, TM_{11}, or TE_{11} waves. They will couple to TE_{20} and TE_{01} modes. One may then choose the slot separation l and the secondary guide phase constant β_0 to reject either the TE_{01} or TE_{20} mode. The coupler will then couple to only one mode out of the five possible in the primary guide. Unfortunately, there is no guarantee that one will be able to eliminate all the undesired modes in any given case. It may also be noted that the couplers may produce mode conversions in the primary waveguide. In the example above the longitudinal slots will couple TE_{01} into TE_{20} or TE_{20} into TE_{01} in the primary guide as well as coupling either of these modes into TE_{10} in the secondary guide. However, this mode conversion will be a minor effect as long as the total power coupled from each mode is small.

Multihole Couplers

Following standard coupler theory [6] it can be shown that if the coupler has n coupling apertures whose coupling coefficients have a binomial distribution, then (4) can be written as:

$$|E_{f1}| = 2k_1M_1 \cos^{(n-1)} \frac{(\beta_1 - \beta_0)l}{2} \tag{12}$$

$$|E_{f2}| = 2k_2M_2 \cos^{(n-1)} \frac{(\beta_2 - \beta_0)l}{2}. \tag{13}$$

Substituting into (10), the equation for mode selectivity:

$$S = 20\,(n-1) \log \left[\frac{\cos \dfrac{(\beta_1 - \beta_0)l}{2}}{\cos \dfrac{(\beta_2 - \beta_0)l}{2}} \right] + 20 \log \frac{k_1}{k_2}. \tag{14}$$

It can be seen that the mode selectivity can be increased by using multihole couplers.

Negatively Traveling Waves

The coupling equations, (4) and (6), were derived on the assumption that the signal was traveling to the right. A wave traveling to the left will result in an output:

$$E_{f1}' = k_1M_1'e^{-j\beta_0 l_1}[1 + e^{-j(\beta_0+\beta_1)l}] \tag{15}$$

for M_1 or

$$|E_{f1}'| = k_1 M_1' \cos \frac{(\beta_0 + \beta_1)l}{2} \quad (16)$$

and

$$|E_{f2}| = k_2 M_2' \cos \frac{(\beta_0 + \beta_2)l}{2} \quad (17)$$

for M_2'. It is apparent that in general both of the backward traveling waves will appear at the output terminal. Since this will seriously interfere with the measurement of the forward wave, reflections from the load must be held to a minimum.

Error Considerations

In using the couplers one may simply ignore the effects of the cross-coupling when making a set of measurements. Since the mode selectivities cannot be infinite, this procedure will result in some error. A relationship between the mode selectivity and this error is of considerable interest.

To develop the error equation we start with (2). The first equation is divided by k_{11}, the second by k_{22}, and so on, giving:

$$\begin{aligned}
\frac{E_1}{k_{11}} &= M_1 + \frac{k_{12}M_2}{k_{11}} + \cdots \frac{k_{1j}}{k_{11}} + \cdots \frac{k_{1n}}{k_{11}} M_n \\
\frac{E_2}{k_{22}} &= \frac{k_{21}}{k_{22}} M_1 + M_2 + \cdots \\
&\vdots \\
\frac{E_r}{k_{rr}} &= \frac{k_{r1}}{k_{rr}} M_1 + \cdots \quad \frac{k_{rj}}{k_{rr}} M_j \cdots \frac{k_{rn}}{k_{rr}} M_n \\
&\vdots \\
\frac{E_n}{k_{nn}} &= \frac{k_{n1}}{k_{nn}} M_1 + \quad \cdots \quad M_n.
\end{aligned} \quad (18)$$

Now E_r/k_{rr} is the apparent value of M_r obtained with the rth coupler when the cross-coupling is neglected. For a 5-mode system, the error in the over-all power measurement which will result when the interference is neglected will be:

$$\delta \text{ db} = 10 \log_{10} \frac{\sum_{r=1}^{5} \left| \frac{E_r}{k_{rr}} \right|^2}{\sum_{j=1}^{5} |M_j|^2}. \quad (19)$$

This states that the error δ is 10 times the log of the ratio of the apparent power to the true power. Substituting from (18) this becomes:

$$\delta = 10 \log_{10} \frac{\sum_{r=1}^{5} \left[\sum_{j=1}^{5} M_j \frac{k_{rj}}{k_{rr}} \right]^2}{\sum_{j=1}^{5} |M_j|^2}. \quad (20)$$

The maximum error will occur when the coupling coefficients k_{rr} have the same value for all r, when the cross-coupling coefficients k_{rj} have the same value for all r and j, and when the modes are of equal amplitude. Eq. (20) then reduces to:

$$\delta \text{ db} = 20 \log_{10} \left[1 + 4 \left| \frac{k_{rj}}{k_{rr}} \right| \underline{/\psi} \right] \quad (21)$$

or, setting $|k_{rj}/k_{rr}| \underline{/\psi} = S$,

$$\delta \text{ db} = 20 \log (1 + 4s). \quad (22)$$

Since s is complex, the maximum value of δ will be obtained when s is a negative real number.

$$\delta \max = 20 \log (1 - 4|s|). \quad (23)$$

If the s values and M's are different instead of being equal, the situation is more complicated, but it can be shown that the maximum total error will always be less than δ max [assuming of course that the maximum value of s is equal to the value of s in (22)].

A more realistic case is obtained by assuming that for each coupler only one mode causes serious interference, and further that each mode causes interference only once during a set of measurements. If all the s's are again equal, then:

$$\delta \max = 20 \log (1 - s). \quad (24)$$

These two curves of maximum error are plotted in Fig. 6 as a function of S, the mode selectivity where:

$$S = 20 \log s. \quad (25)$$

The needed mode selectivity for any desired degree of accuracy can be obtained directly from these curves. In practice, the error will usually be much less than indicated by Fig. 6, since this represents the worst possible case.

The error curves of Fig. 6 can be used to specify the return loss required of the load. Eqs. (16) and (17) show that in the worst possible case the reflected waves will be coupled exactly as strongly as the desired incident mode. If the return loss for a given mode is 20 db, then the effective mode selectivity for that mode will be 20 db. Therefore, one may relabel the S axis of Fig. 6 as return loss and obtain a direct relation between the quality of the load and the worst error it can introduce.

If the amplitudes of the individual modes are to be found, the mode selectivity requirements become more severe. Reference to (18) shows that the error depends on the relative strength of each mode as well as the mode selectivity of each coupler. Fortunately, the largest error is always made in connection with the

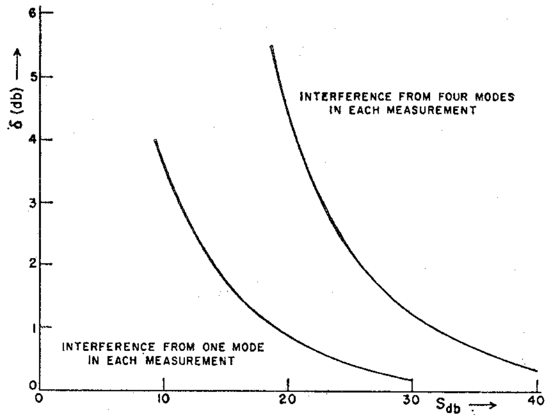

Fig. 6—The maximum error in the measurement of total power when the mode interference is ignored.

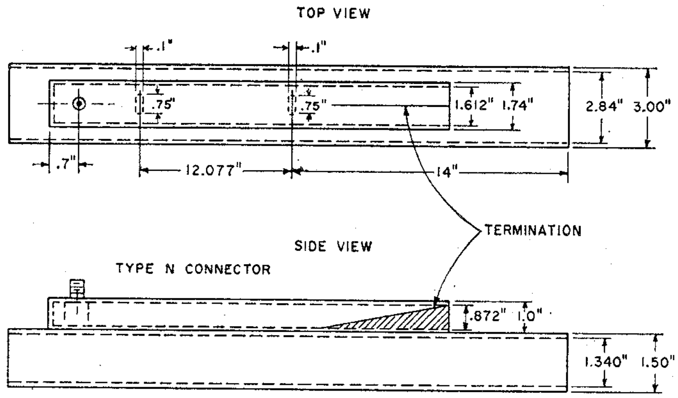

Fig. 7—TE_{10} coupler.

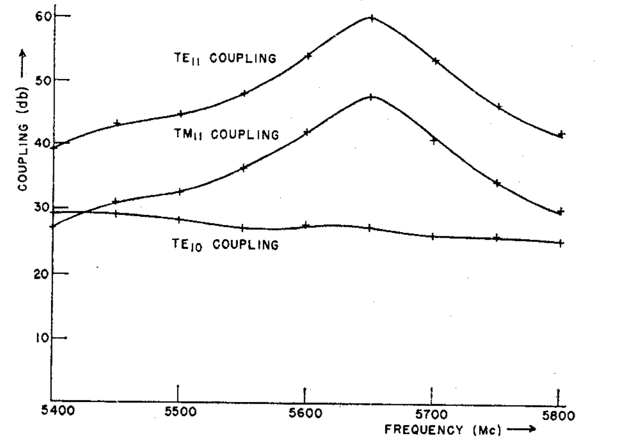

Fig. 8—TE_{10} coupler. Experimental curves of coupling as a function of frequency for the TE_{10}, TE_{11}, and TM_{11} modes. Coupling to the TE_{01} and the TE_{20} modes was less than 70 db.

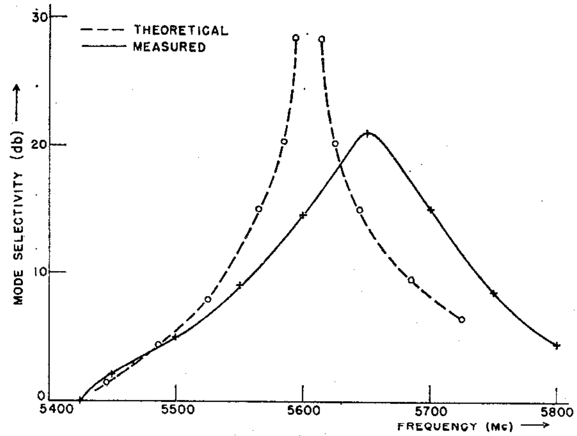

Fig. 9—TE_{10} coupler. Mode selectivity as a function of frequency for the TM_{11} mode.

measurement of the weakest and therefore least significant mode.

Experimental Models

Three typical mode couplers are shown in Figs. 7 through 15. In each case the primary guide was a length of standard *S*-band waveguide. The secondary guide is fabricated from sheet brass. Note that because the secondary guide is operated much closer to cutoff than normal, special attention must be given to the design of the probes, terminations, and bends used in this guide. The coupling aperture used in the couplers is a slot 0.75 inch long by 0.1 inch wide.

The curves of coupling and mode selectivity show rather good agreement with the theoretical predictions. Mode selectivities ranging from 15 db to greater than 30 db were obtainable over a 200-mc bandwidth. The frequency of maximum selectivity for the couplers was within 1 per cent (or about 50 mc) of the design frequency in every case.

Application of the Couplers

It has been pointed out that for second harmonic measurements five couplers, one for each mode, will be required. For third harmonic measurements one would evidently require 11 couplers, and so on, with the required number of couplers increasing rapidly with the order of the harmonic. Clearly a point is very quickly reached where the complexity and expense of such a system would make practical application out of the question. Furthermore, with the techniques outlined thus far, there is no way of rejecting an arbitrarily large number of modes. The problem is further complicated by the existence of degenerate modes. These modes can only be separated by means of aperture selectivity, since mode selectivity based on differences in phase constants obviously cannot be obtained.

Fortunately, the number of modes which actually propagate is often considerably less than the number theoretically possible. As an example, an idea symmetrical coax to waveguide adaptor (consisting of a waveguide with a perpendicular probe in the center of the broad face) when operating at a second harmonic frequency excites TE_{11}, TM_{11}, and TE_{10}, but does not excite TE_{20} and TE_{01} although both of these latter modes could propagate. This is due to the mechanical symmetry of the device. Only three couplers will be required to measure the second harmonic modes excited by such a probe. Since this type of adaptor is common in microwave circuits, these three couplers will handle a wide variety of such specific problems as the

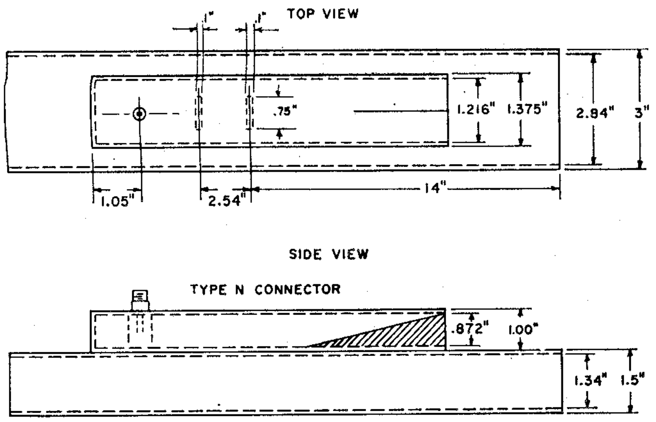

Fig. 10—TM_{11} coupler.

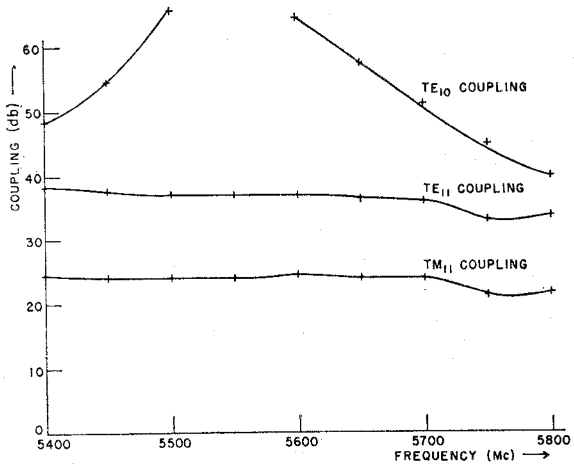

Fig. 11—TM_{11} coupler. Experimental curves of coupling as a function of frequency for the TM_{11}, TE_{11}, and TE_{10} modes. Coupling to the TE_{01} and TE_{20} modes was less than 70 db.

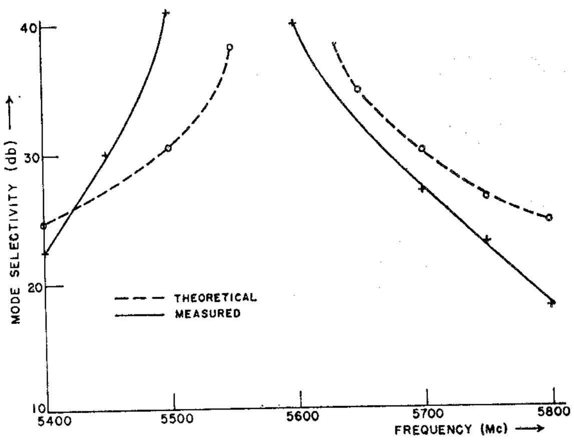

Fig. 12—TM_{11} coupler. Mode selectivity as a function of frequency for the TE_{10} modes.

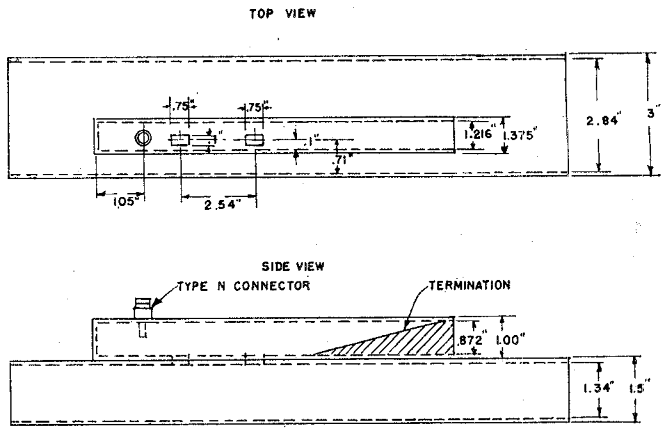

Fig. 13—TE_{11} coupler.

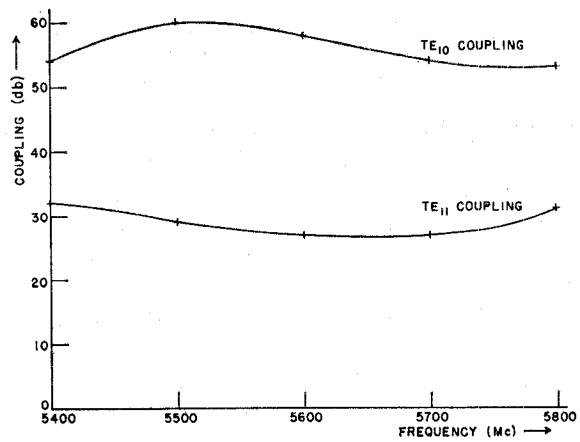

Fig. 14—TE_{11} coupler. Experimental curves of coupling as a function of frequency for the TE_{11} and TE_{10} modes. Coupling to the TM_{11} and TE_{20} modes was less than 70 db. Coupling to the TE_{01} mode was approximately constant at 30 db.

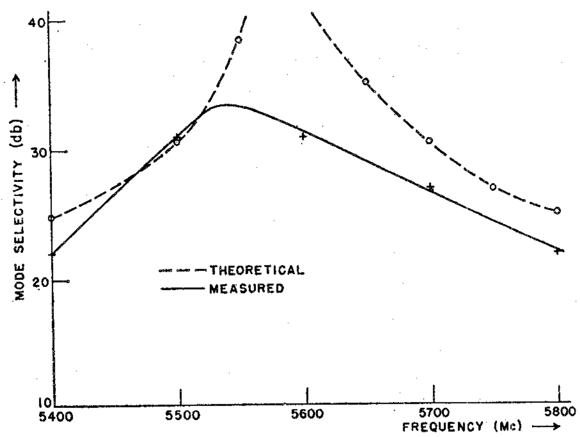

Fig. 15—TE_{11} coupler. Mode selectivity as a function of frequency for the TE_{10} mode.

measurement of the second harmonic output of magnetrons. The three couplers of Figs. 7 through 15 were designed for this application.

The results of a series of second harmonic measurements on an RK5586 magnetron with a set of five couplers are shown in Fig. 16. These curves were obtained by measuring the power in each mode as the magnetron supply voltage was varied. The magnetron was operated at 2800 mc at a pulse repetition rate of 1000 PPS with a 1-μsec pulse length. The termination used was a standard *S*-band dummy load which had a VSWR ranging from less than 1.05 for the TE_{10} mode to a maximum of 1.1 for the TE_{11} mode. The measurements were made directly at the output of the coax to waveguide trans-

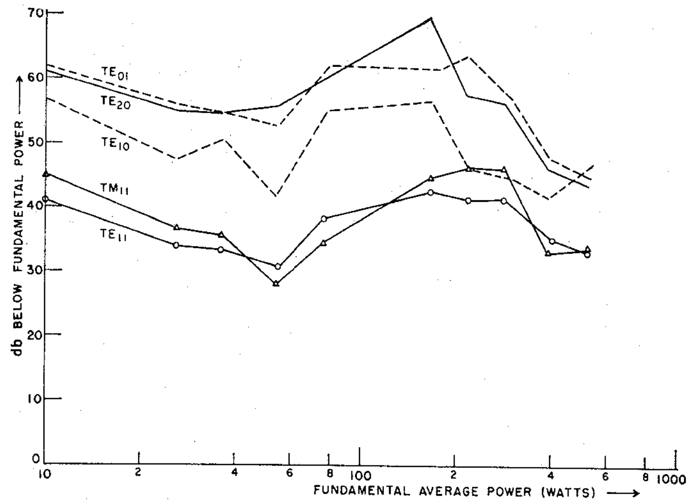

Fig. 16—Second harmonic average power content in the TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11} modes.

ducer used with the 5586 magnetron. Evidently the concentration of power in the TE_{11}, TM_{11}, and TE_{10} modes is due to the symmetry of this transducer.

The couplers used to obtain the curves of Fig. 16 were adapted for use in a pressurized system. However, at the power levels involved (500 kw) arcing was not a problem and pressurization was not used. These measurements were made by Dr. Pietro Lombardini [7].

Conclusions

Experimental models of the mode couplers indicate that mode selectivities of the order of 30 db or greater over bandwidths of 200 mc or greater are obtainable in simple two-hole couplers. Degenerate modes pose a special problem and may result in substantially lower selectivities. As many as four modes can be simultaneously rejected while coupling strongly to a fifth. The application of these couplers seems limited to systems in which the number of propagating modes is small, but within this limitation they provide a quick and covenient power measurement technique.

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Measurement of Harmonic Power Generated by Microwave Transmitters*

VERNON G. PRICE†

Summary—A measurement technique is described that can be used to determine quantitatively the power levels of the higher order modes propagating in a straight, lossless, rectangular waveguide. The technique employs a number of small calibrated electric probes which are fixed on the broad and narrow walls of the waveguide measurement section to sample the electric fields within. The method used to calibrate these probes is briefly discussed, and information on accuracy and limitations of the probe technique is presented. Some measurement results on the power levels in the modes of the second and third harmonic frequencies in the outputs of high power *S*-band magnetrons and klystrons are presented.

The multiple-probe technique has reduced the time required to take measurements at a given frequency to about one-half hour. An automatic computer has been programmed to perform all of the required mathematical operations and has reduced the computation time to less than one-half hour for each measurement frequency.

Introduction

In recent years several workers have advanced methods of measurement of the harmonic frequency power in the output of microwave tubes. Interest in these measurements has been stimulated to a large extent by the acute problems of radio interference between microwave systems which can result from unwanted radiation of harmonic frequency energy. However, measurement of this power is complicated by the fact that harmonics in waveguide lines may propagate in many different modes which are convertible one to another by the presence of obstacles or ports. This paper

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